Analog Communications

— Lecture 04 —

Performance of Linear Modulation over AWGN Channel

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Outline

1. Transmitted and Received Signals
2. Performance Indicators
3. AM and Envelope Detector
4. Graphical Representation

Transmitted Signal

The transmitted signal in the case of linear modulation is

\[ z(t) = z_I(t) \cos(2\pi f_0 t) + z_Q(t) \sin(2\pi f_0 t) \]

where

- DSB: \( z_I(t) = Ax(t) \) and \( z_Q(t) = 0 \)
- AM: \( z_I(t) = A(1 + kx(t)) \) and \( z_Q(t) = 0 \)
- SSB: \( z_I(t) = Ax(t) \) and \( z_Q(t) = \mp A\hat{x}(t) \)
- QAM: \( z_I(t) = Ax_1(t) \) and \( z_Q(t) = Ax_2(t) \)

Denote

- \( B_x \) the bandwidth of the information signal
- \( B_z \) the band of the transmitted signal

AWGN Channel

Assumptions:

- \( H_T(f)H_C(f)H_R(f) \) is flat over \( B_z \)
- \( w_o(t) \) is white and Gaussian
Received Signal over AWGN

The received signal in the case of AWGN channel is
\[
r(t) = z(t) + w(t)
\]
where
\[
P_w(f) = \begin{cases} \frac{\eta_o}{2} & |f| \in B_z \\ 0 & |f| \notin B_z \end{cases}
\]
and
- DSB, AM, QAM: \( B_z = (f_0 - B_x, f_0 + B_x) \)
- SSB: \( B_z = (f_0 - B_x, f_0) \) or \( B_z = (f_0, f_0 + B_x) \)

Coherent Receiver

The received signal is a band-pass signal
(being the sum of two band-pass signals)
\[
r(t) = z(t) + w(t)
= r_I(t) \cos(2\pi f_0 t) + r_Q(t) \sin(2\pi f_0 t)
\]
where
\[
r_I(t) = z_I(t) + w_I(t)
\]
\[
r_Q(t) = z_Q(t) + w_Q(t)
\]
A coherent receiver (in case of perfect synchronization) provides
\[
y(t) = \frac{B}{2} r_I(t)
= \frac{B}{2} z_I(t) + \frac{B}{2} w_I(t)
\]

Performance (1/3)

The Signal-to-Noise Ratio (SNR) at the input of the receiver is
\[
\text{SNR}_{in} = \frac{P_z}{P_w}
\]

DSB:
\[
P_z = \frac{A^2}{2} P_x, \quad P_w = 2\eta_o B_x, \quad \text{SNR}_{in} = \frac{A^2 P_x}{4\eta_o B_x}
\]
AM:
\[
P_z = \frac{A^2}{2} (1 + k^2 P_x), \quad P_w = 2\eta_o B_x, \quad \text{SNR}_{in} = \frac{A^2 (1 + k^2 P_x)}{4\eta_o B_x}
\]
SSB:
\[
P_z = \frac{A^2 P_x}{\eta_o B_x}, \quad P_w = \eta_o B_x, \quad \text{SNR}_{in} = \frac{A^2 P_x}{\eta_o B_x}
\]
QAM:
\[
P_z = \frac{A^2}{2} (P_{x_1} + P_{x_2}), \quad P_w = 2\eta_o B_x, \quad \text{SNR}_{in} = \frac{A^2 (P_{x_1} + P_{x_2})}{4\eta_o B_x}
\]

Performance (2/3)

The Signal-to-Noise Ratio (SNR) at the output of the receiver is
\[
\text{SNR}_{out} = \frac{\frac{B^2 P_z}{4} P_{z_I}}{\frac{B^2}{4} P_{w_I}} = \frac{P_{z_I}}{P_{w_I}}
\]

DSB:
\[
P_{z_I} = A^2 P_x, \quad P_{w_I} = 2\eta_o B_x, \quad \text{SNR}_{out} = \frac{A^2 P_x}{2\eta_o B_x}
\]
AM:
\[
P_{z_I} = A^2 P_x, \quad P_{w_I} = 2\eta_o B_x, \quad \text{SNR}_{out} = \frac{A^2 k^2 P_x}{2\eta_o B_x}
\]
SSB:
\[
P_{z_I} = A^2 P_x, \quad P_{w_I} = \eta_o B_x, \quad \text{SNR}_{out} = \frac{A^2 P_x}{\eta_o B_x}
\]
QAM:
\[
P_{z_{I_1}} = A^2 P_{x_1}, \quad P_{w_I} = 2\eta_o B_x, \quad \text{SNR}_{out} = \frac{A^2 P_{x_1}}{2\eta_o B_x}
\]
Performance (3/3)

Common perf. indicators are $\text{SNR}_{\text{out}}(\text{SNR}_{\text{in}})$ and $\text{SNR}_{\text{out}}(\gamma)$ where

$$\gamma = \frac{P_s}{\eta_i B_x}$$

represents the ratio of two powers

- the power of the transmitted signal at the numerator
- the product between the noise PSD and the information bandwidth

<table>
<thead>
<tr>
<th>Modulation</th>
<th>$\text{SNR}_{\text{out}}$</th>
<th>$\text{SNR}_{\text{in}}$</th>
<th>$\text{SNR}_{\text{out}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>DSB</td>
<td>$2 \ \text{SNR}_{\text{in}}$</td>
<td>$\text{SNR}_{\text{in}}$</td>
<td>$\gamma$</td>
</tr>
<tr>
<td>AM</td>
<td>$\frac{2}{1 + \frac{I_2}{2P_x}} \ \text{SNR}_{\text{in}}$</td>
<td>$\frac{1}{1 + \frac{I_2}{2P_x}} \ \gamma$</td>
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</tr>
<tr>
<td>SSB</td>
<td>$\text{SNR}_{\text{in}}$</td>
<td>$\text{SNR}_{\text{in}}$</td>
<td>$\gamma$</td>
</tr>
<tr>
<td>QAM</td>
<td>$\text{SNR}<em>{\text{out},i} = \text{SNR}</em>{\text{in},i}$</td>
<td>$\text{SNR}_{\text{out}} = \gamma_i$</td>
<td></td>
</tr>
</tbody>
</table>

AM reception with Envelope Detector - low SNR

$w_i^2(t) + w_Q^2(t)$ is the dominant term within the square root, then

$$y(t) = \sqrt{w_i^2(t) + w_Q^2(t)} \sqrt{A^2(1 + kx(t))^2 + 2A(1 + kx(t))w_i(t) + w_Q^2(t) + 1}$$

$$\approx \sqrt{w_i^2(t) + w_Q^2(t)} \left( 1 + \frac{A(1 + kx(t))w_i(t)}{w_i^2(t) + w_Q^2(t)} \right)$$

$$\approx \rho(t) + A(1 + kx(t)) \cos(\theta(t))$$

The term $\cos(\theta(t))$ affects irremediably the useful signal

The whole system does NOT work

AM reception with Envelope Detector - high SNR

$A(1 + kx(t))$ is the dominant term within the square root, then

$$y(t) = A(1 + kx(t)) \sqrt{1 + 2 \frac{w_i(t)}{A(1 + kx(t))} + \frac{w_i^2(t) + w_Q^2(t)}{A^2(1 + kx(t))^2}}$$

$$\approx A(1 + kx(t)) \left( 1 + \frac{w_i(t)}{A(1 + kx(t))} \right)$$

$$\approx A(1 + kx(t)) + w_i(t)$$

Same performance as for the coherent receiver
Graphical Representation for DSB

Graphical Representation for AM (coherent receiver)

Graphical Representation for SSB

Graphical Representation for QAM
Graphical Representation for AM (envelope detector)

Graphical Representation for Phase Error